Code No: 131AA **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, March/April - 2023
MATHEMATICS - I
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Define an exact differential equation. Write the solution of that exact differential-equation. [2]
b) Write the general solution of $(D^2+a^2)\,y = \sin ax$. [3]
c) Define a Hermitian matrix and a skew-Hermitical matrix. [2]
d) Define Echelon form of a matrix. What is the rank of a matrix which is in Echelon form. [3]
e) Show that the sum of the eigen values is equal to its trace. [2]
f) Define a quadratic form and give its real symmetric matrix. [3]
g) If $z = e^{xy}$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ [2]
h) If $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(r,\theta)}{\partial(x,y)}$. [3]
i) Form a partial differential equation from $z = f(x^2+y^2)$ by the elimination of arbitrary function. [2]
j) Solve $px + qy = z$. [3]

PART - B

(50 Marks)

2.a) Solve $\left(\cos x\tan y + \cos(x+y)\right)dx + \left(\sin x\sec^2 y + \cos(x+y)\right)dy = 0$.
b) According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 30 ^{0}C and the substance cools from 100 ^{0}C to 70^{0}C in 15 minutes, find when the temperature will be 40^{0}C? [5+5]

OR

3. Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by method of variation of parameters. [10]

4.a) Find the rank of the matrix A = $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.

b) Solve the system of equation $3x+3y+2z=1;\ x+2y=4;\ 10y+3z=-2;\ 2x-3y-z=5$. [5+5]

OR

5. Solve the system $\lambda x+y+z=0;\ x+\lambda y+z=0;\ x+y+\lambda z=0$ for all values of λ, if it has a non-trivial solution. [10]

6. Verify Cayley-hamilton theorem for the matrix A = $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the value of $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$. [10]

OR

7. Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the canonical form and find its nature. [10]

8.a) If $u=\log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$.

b) If $u=\sin^{-1}(x-y),\ x=3t,\ y=4t^3$, show that $\frac{du}{dt}=\frac{3}{\sqrt{1-t^2}}$. [5+5]

OR

9. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. [10]

10.a) Form a partial differential equation from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

b) Solve $\left(x^2-y^2-z^2\right)p+2xyq=2zx$. [5+5]

OR

11.a) For a partial differential equation from $f(x+y+z, x^2+y^2+z^2)=0$.

b) Solve $p^2-q^2=x-y$. [5+5]

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